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SHORTER COMMUNICATIONS

HEAT TRANSFER IN FULLY DEVELOPED LAMINAR FLOW THROUGH RECTANGULAR AND ISOSCELES TRIANGULAR DUCTS

F. W. SCHMIDT and M. E. NEWELL
Mechanical Engineering Department, Imperial College, London

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NOMENCLATURE

AR, aspect ratio (B/A);
D_h, characteristic length $\left(\frac{4 \text{ cross sectional area}}{\text{heated perimeter}}\right)$;
h, heat-transfer coefficient;
k, thermal conductivity;
Nu, Nusselt number $\left(\frac{hD_h}{k}\right)$;
q', heat flux per unit length of duct;
T, $\frac{t_0 - t}{t_0 - t_m}$;
t, temperature;
t_m, mixing cup temperature $\left(\frac{\iint tv \, dx \, dy}{\iint v \, dx \, dy}\right)$;
t_0, wall temperature;
v, velocity;
v_m, mean bulk velocity;
α, apex half angle.

THE OBJECT of this communication is to present additional values of the Nusselt number for fully developed laminar flow through rectangular and isosceles triangular ducts with various aspect ratios. Two types of boundary conditions were investigated: constant wall temperature and constant heat flux per unit length with a uniform wall temperature for the conducting surfaces at each cross section. Results were also obtained for configurations where one or more of the walls were adiabatic. The following assumptions have been made:

(a) velocity and temperature profiles are fully developed, $v = f(x, y)$ and $T = f(x, y)$;
(b) constant fluid properties;
(c) no viscous dissipation;
(d) natural convection effects are negligible, and
(e) negligible heat conduction in the axial direction.

Several papers have been published presenting heat-transfer results for fully developed laminar flow in ducts. For example, the case of a rectangular duct with all sides conducting was presented by Clark and Kays [1]; also, the triangular-duct case with all sides conducting and constant heat flux per unit length, was reported by Sparrow and Haji-Sheikh [2]. Both of these papers utilized finite-difference techniques for the solution of the momentum and energy differential equations.

The rectangular configuration in this study, Fig. 1, was subdivided into a rectangular system of grids. The triangular configuration, after being normalized, was subdivided into a square system of grids, thus allowing all the walls to pass through grid intersections. The normalized momentum and energy equations were then represented at each node, i.e. grid intersection, by difference equations found using finite difference techniques. The resulting system of linear algebraic equations was solved on an IBM 7090 digital computer using a subroutine employing a Gaussian elimination technique followed by an iterative double precision arithmetic correction of the solution to improve its accuracy. In order to use the finest grid spacing in the computations, symmetry was used whenever possible. The rectangular duct was divided into a system of 20 subdivisions in the long direction and 10 in the short when one axis of symmetry was present and 20 × 20 subdivisions when symmetry was present about both axes. A system of 10 × 10 subdivisions was used when symmetry was not present requiring that the complete duct be considered. The triangular duct was subdivided into a system of 28 in the x direction by 14 in the y direction when symmetry was present, but when the complete duct was considered 10 subdivisions were used in the y direction and 20 in the x direction. The bulk velocity, v_m, and the mixing cup temperature, t_m, were found using a two-dimensional extension of Simpson's rule.

To assist in the evaluation of the accuracy of the procedure used, especially the iterative method required when solving the energy equation in the constant wall temperature case, the fully developed infinite parallel plate problem was solved using finite difference techniques. The iterative method was necessary since dT/dZ was a function of x and y for these boundary conditions in the rectangular duct while only a function of y in the infinite parallel plate case. The results and a comparison with analytical results are presented in Table 1. A significant result was obtained for the case of constant wall temperature. The calculated value fell between the two previously reported values.

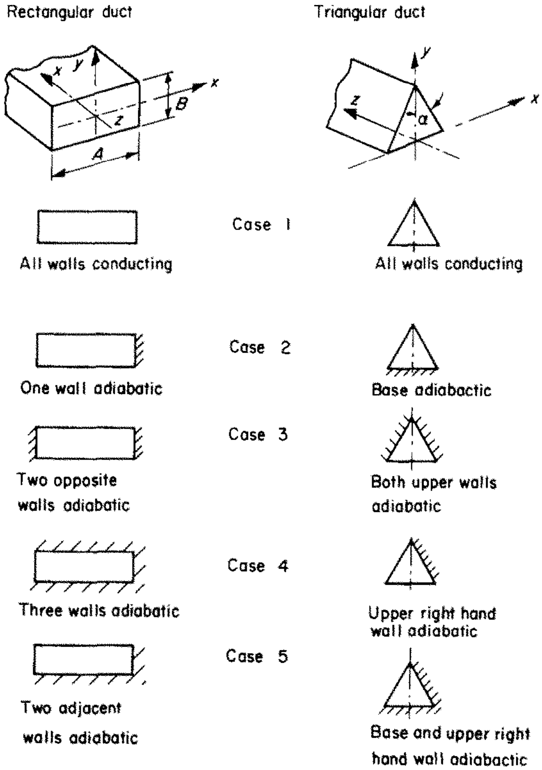

FIG. 1. Duct configuration.

Table 1. Nusselt numbers for infinite parallel plates

Boundary condition		Both sides conducting	One side conducting One side adiabatic
q/A = constant	analytical	8·2353	10·7692
	numerical	8·2351	10·7696
t_0 = constant	analytical	7·50 [3], 7·60 [4]	9·72 [5]
	numerical	7·5405	9·7221

An analytical expression for the velocity profile in a rectangular duct was presented by Savino *et al.* [6]. As expected, comparison with our work indicated that the error was a function of aspect ratio and location. The maximum error found for v/v_m was encountered with an aspect ratio of 0·1 and was less than 0·85 per cent. Most values were less than 0·1 per cent in error. Sparrow [7] presented an analytical solution for the velocity distribution in a triangular duct. Comparison of v/v_m along the line of symmetry for a triangular duct with an included angle of 40° indicated agreement to within four significant figures. Since the coefficients tabulated in the paper were stated to be accurate to only four significant figures, the agreement is very good.

The results for the rectangular duct are shown in Fig. 2. The values for Nu obtained by Clark and Kays [1] are indicated on the curves for case 1 and show good agreement with our results. Results for the limiting aspect ratio, $AR \to 0$ and $AR \to \infty$, were not calculated using the rec-

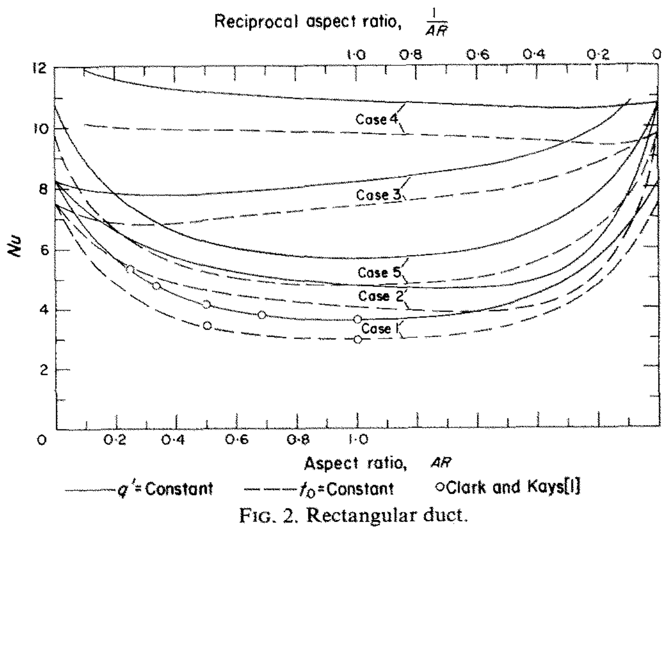

FIG. 2. Rectangular duct.

tangular duct computer program. However, as we approached these limits the configuration approached those of parallel infinite plates. In case 4, as AR tends to zero while B remains finite, Nu tends to infinity, because D_h tends to infinity.

In the triangular duct results shown in Fig. 3, the limiting cases are not as clearly indicated. For the case of constant q', Sparrow and Haji-Sheikh [2] indicated that the limiting Nu would be one-quarter of that obtained in the appropriate infinite plate solution. The results for our calculations, case 1, q' = constant, agreed closely with those presented by Sparrow and Haji-Sheikh [2] and are shown in Fig. 3.

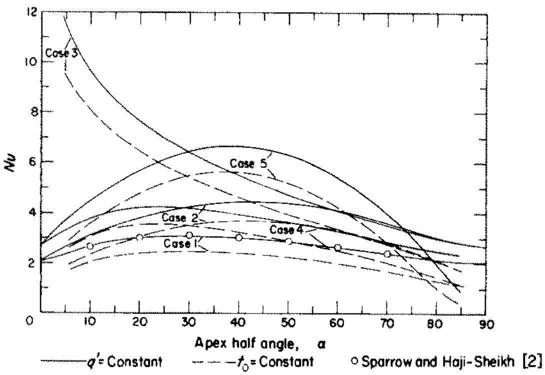

FIG. 3. Triangular duct.

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A METHOD OF CORRELATING LOCAL AND AVERAGE FRICTION COEFFICIENTS FOR BOTH LAMINAR AND TURBULENT FLOW OF GASES THROUGH A SMOOTH TUBE WITH SURFACE TO FLUID BULK TEMPERATURE RATIOS FROM 0·35 TO 7·35

MAYNARD F. TAYLOR
Lewis Research Center, Cleveland, Ohio

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NOMENCLATURE

D, inside diameter of test section;
f, fanning friction coefficient;
L, test section length;
Re, Reynolds number;
T_b, bulk stagnation temperature;
T_f, film temperature $(T_s + T_b)/2$;
T_s, surface temperature;
X, distance from entrance of test section.

Subscript
s, denotes physical properties evaluated at the surface temperature.

INTRODUCTION

THERE has long been a need of a means of correlating both laminar and turbulent friction coefficients for gases with large variations in the physical properties flowing through smooth tubes. Probably the most widely used method of correlating and predicting friction coefficients for turbulent